

# The more investigation about inflation and reheating stages based on the Planck and WMAP-9

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## Abstract

The potential  $V(\phi) = \lambda\phi^n$  is responsible for the inflation of the universe as scalar field  $\phi$  oscillates quickly around some point where  $V(\phi)$  has a minimum. The end of this stage has an important role on the further evolution stages of the universe. The created particles are responsible for reheating the universe at the end of this stage. The behaviour of the inflation and reheating stages are often known as power law expansion  $S(\eta) \propto \eta^{1+\beta}$ ,  $S(\eta) \propto \eta^{1+\beta_s}$  respectively. The reheating temperature ( $T_{rh}$ ) and  $\beta_s$  give us valuable information about the reheating stage. Recently people have studied about the behaviour of  $T_{rh}$  based on slow-roll inflation and initial condition of quantum normalization. It is shown that there is some discrepancy on  $T_{rh}$  due to amount of  $\beta_s$  under the condition of slow-roll inflation and quantum normalization [6]. Therefore the author is believed in [6] that the quantum normalization may not be a good initial condition. But it seems that we can remove this discrepancy by determining the appropriate parameter  $\beta_s$  and hence the obtained temperatures based on the calculated  $\beta_s$  are in favour of both mentioned conditions. Then from given  $\beta_s$ , we can calculate  $T_{rh}$ , tensor to scalar ratio  $r$  and parameters  $\beta, n$  based on the Planck and WMAP-9 data. The observed results of  $r, \beta_s, \beta$  and  $n$  have consistency with their constrains. Also the results of  $T_{rh}$  are in agreement with its general range and special range based on the DECIGO and BBO detectors.

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## I. INTRODUCTION

The potential  $V(\phi) = \lambda\phi^n$  with constant  $\lambda$  is responsible for the inflation of the universe as scalar field  $\phi$  oscillates quickly around some point where  $V(\phi)$  has a minimum. The end of this stage has an important role on the further evolution stages of the universe. The created particles are responsible for reheating the universe at the end of this stage. Because of the inflation stage brought temperature of the universe below for the requirement of thermo nuclear reactions, therefore the reheating stage was necessary for the nucleosynthesis process. The people have many studies about the reheating process but until now it is not well-known. In the literatures [1] [2] [3] the behaviour of the inflation and reheating stages are often known as power law expansion  $S(\eta) \propto \eta^{1+\beta}$ ,  $S(\eta) \propto \eta^{1+\beta_s}$  respectively, where  $S(\eta)$  and  $\eta$  are scale factor and conformal time respectively. The parameters  $\beta$  and  $\beta_s$  have constrained to  $1 + \beta < 0$ ,  $1 + \beta_s > 0$  [1, 4]. These stages have important effect on the evolution of relic gravitational waves (RGWs).

The reheating temperature ( $T_{rh}$ ) with general range  $10 \text{ MeV} \lesssim T_{rh} \lesssim 10^{16} \text{ GeV}$  [5] and  $\beta_s$  are very important and give us valuable information about the reheating stage. Recently people have studied about the behaviour of  $T_{rh}$  based on the slow-roll inflation and initial condition of the quantum normalization [6]. It is shown that there is some discrepancy on  $T_{rh}$  due to amount of  $\beta_s$  under the slow-roll inflation and quantum normalization [6]. In case of the slow-roll inflation is the correct condition, then the author has said in [6] that “the quantum normalization may not be a good initial condition”. Note that, if we do not consider quantum normalization, the zero point energy must be removed or else the cosmological constant would be 120 orders of magnitude more than observed. Therefore based on this point, we think that one can remove this discrepancy by determining of the appropriate parameter  $\beta_s$  provided it dose not exceed the condition  $1 + \beta_s > 0$ , and also the obtained spectral energy density is not more than level  $\simeq 10^{-6}$  due to  $\beta_s$  [1]. Hence the obtained temperatures based on the calculated  $\beta_s$  are in favour of both slow roll and initial condition of quantum normalization. Thus the main purpose of this work is removing the mentioned discrepancy by estimating the appropriate parameter  $\beta_s$ . Then from estimated  $\beta_s$ , we calculate  $T_{rh}$ , tensor to scalar ratio  $r$ , and parameters  $\beta, n$  based on the WMAP-9 and Planck 2015 data [7, 8]. The observed results of  $r, \beta_s, \beta$  and  $n$  have consistency with their constrains. Also the results of  $T_{rh}$  are in agreement with its general range and special range

$10^6 \text{ GeV} \lesssim T_{rh} \lesssim 10^9 \text{ GeV}$  [9] based on the DECIGO-hertz Interferometer Gravitational-wave Observatory (DECIGO) [10, 11] and the Big-Bang Observer (BBO) [12]. Finally we obtain the interesting reheating temperature  $10^7 \text{ GeV}$  in our results as this temperature is best determined for  $r \sim 0.1$  based on the DECIGO and BBO detectors [13]. Hence based on the results in this work, we can have better understanding about the history of the universe especially inflation and reheating stages. We use the units  $c = \hbar = k_B = 1$ .

## II. SPECTRUM OF GRAVITATIONAL WAVES

The perturbed metric for a homogeneous isotropic flat Friedmann-Robertson-Walker universe can be written as

$$ds^2 = S^2(\eta)(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j), \quad (1)$$

where  $S(\eta)$ ,  $\eta$  and  $\delta_{ij}$  are scale factor, conformal time and Kronecker delta respectively. The  $h_{ij}$  are metric perturbations field contain gravitational waves with transverse-traceless properties i.e;  $\nabla_i h^{ij} = 0$ ,  $\delta^{ij} h_{ij} = 0$ .

This work assumes the shape of the spectrum of RGWs generated by the expanding space time background. Therefore the perturbed matter source is not taken into account. We can describe gravitational waves with the linearized field equation that given by

$$\nabla_\mu (\sqrt{-g} \nabla^\mu h_{ij}(\mathbf{x}, \eta)) = 0. \quad (2)$$

To compute the  $h(\mathbf{x}, \eta)$ , we consider polarization modes  $h^+$  and  $h^\times$  in terms of the creation ( $a^\dagger$ ) and annihilation ( $a$ ) operators,

$$\begin{aligned} h_{ij}(\mathbf{x}, \eta) &= \frac{\sqrt{16\pi} l_{pl}}{S(\eta)} \sum_{\mathbf{p}} \int \frac{d^3 k}{(2\pi)^{3/2}} \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) \\ &\times \frac{1}{\sqrt{2k}} \left[ a_{\mathbf{k}}^{\mathbf{p}} h_{\mathbf{k}}^{\mathbf{p}}(\eta) e^{i\mathbf{k} \cdot \mathbf{x}} + a_{\mathbf{k}}^{\dagger \mathbf{p}} h_{\mathbf{k}}^{*\mathbf{p}}(\eta) e^{-i\mathbf{k} \cdot \mathbf{x}} \right], \end{aligned} \quad (3)$$

where  $\mathbf{k}$  is the comoving wave number,  $k = |\mathbf{k}|$ ,  $l_{pl} = \sqrt{G}$  is the Planck's length and  $\mathbf{p} = +, \times$  are polarization modes. The polarization tensor  $\epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$  is symmetric and transverse-traceless  $k^i \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) = 0$ ,  $\delta^{ij} \epsilon_{ij}^{\mathbf{p}}(\mathbf{k}) = 0$  and satisfy the conditions  $\epsilon^{ij\mathbf{p}}(\mathbf{k}) \epsilon_{ij}^{\mathbf{p}'}(\mathbf{k}) = 2\delta_{\mathbf{p}\mathbf{p}'}$  and  $\epsilon_{ij}^{\mathbf{p}}(-\mathbf{k}) = \epsilon_{ij}^{\mathbf{p}}(\mathbf{k})$ . The annihilation and creation operators satisfy  $[a_{\mathbf{k}}^{\mathbf{p}}, a_{\mathbf{k}'}^{\dagger \mathbf{p}'}] = \delta_{\mathbf{p}\mathbf{p}'} \delta^3(\mathbf{k} - \mathbf{k}')$

and the initial state is defined as

$$a_{\mathbf{k}}^{\mathbf{p}}|0\rangle = 0, \quad (4)$$

for each  $\mathbf{k}$  and  $\mathbf{p}$ .

We can write eq.(2) for a fixed  $\mathbf{k}$  and  $\mathbf{p}$  as follows

$$h_k'' + 2\frac{S'}{S}h_k' + k^2h_k = 0, \quad (5)$$

where prime means derivative with respect to the conformal time. We here onwards denote  $h_k(\eta)$  without the polarization index due to similarity of polarization states.

Then, we change  $h_k(\eta)$  to  $h_k(\eta) = f_k(\eta)/S(\eta)$ , where the mode functions  $f_k(\eta)$  satisfy in the minimally coupled Klein-Gordon equation

$$f_k'' + \left(k^2 - \frac{S''}{S}\right)f_k = 0. \quad (6)$$

The general solution of this equation is a linear combination of the Hankel function with a generic power law for the scale factor  $S = \eta^u$  given by

$$f_k(\eta) = A_k\sqrt{k\eta}H_{u-\frac{1}{2}}^{(1)}(k\eta) + B_k\sqrt{k\eta}H_{u-\frac{1}{2}}^{(2)}(k\eta). \quad (7)$$

We can write an exact solution  $f_k(\eta)$  by matching its value and derivative at the joining points, for of a sequence of successive scale factors with different  $u$  for a given model of the expansion of universe [16].

The history of expansion of the universe can written as follows:

1) Inflation stage:

$$S(\eta) = l_0|\eta|^{1+\beta}, \quad -\infty < \eta \leq \eta_1, \quad (8)$$

where  $1 + \beta < 0$ ,  $\eta < 0$  and  $l_0$  is a constant.

2) Reheating stage:

$$S(\eta) = S_z(\eta - \eta_p)^{1+\beta_s}, \quad \eta_1 < \eta \leq \eta_s, \quad (9)$$

where  $1 + \beta_s > 0$ , see for more details [1].

3) Radiation-dominated stage:

$$S(\eta) = S_e(\eta - \eta_e), \quad \eta_s \leq \eta \leq \eta_2. \quad (10)$$

4) Matter-dominated stage:

$$S(\eta) = S_m(\eta - \eta_m)^2, \quad \eta_2 \leq \eta \leq \eta_E, \quad (11)$$

where  $\eta_E$  is the time when the dark energy density  $\rho_\Lambda$  is equal to the matter energy density  $\rho_m$ .

The value of redshift  $z_E$  at  $\eta_E$  is  $(1 + z_E) = S(\eta_0)/S(\eta_E) \sim 1.3$  for TT, TE, EE+lowP+lensing contribution based on Planck 2015 [8] where  $\eta_0$  is the present time.

5) Accelerating stage:

$$S(\eta) = \ell_0 |\eta - \eta_a|^{-\gamma}, \quad \eta_E \leq \eta \leq \eta_0, \quad (12)$$

where  $\gamma$  is  $\Omega_\Lambda$  dependent parameter, and  $\Omega_\Lambda$  is the energy density contrast. We take  $\gamma \simeq 1.05$  [14] for  $\Omega_\Lambda = 0.692$  from Planck 2015 [8].

For normalization purpose of  $S$ , we put  $|\eta_0 - \eta_a| = 1$  which fixes the  $\eta_a$ , and the constant  $\ell_0$  is fixed by the following relation,

$$\frac{\gamma}{H_0} \equiv \left( \frac{S^2}{S'} \right)_{\eta_0} = \ell_0, \quad (13)$$

where  $\ell_0$  is the Hubble radius at present with  $H_0 = 67.8 \text{ km s}^{-1} \text{Mpc}^{-1}$  from Planck 2015 [8].

The physical wavelength is related to the comoving wave number as  $\lambda \equiv 2\pi S(\eta)/k$ , in the expanding Friedmann- Robertson-Walker space time and the wave number  $k_H$  corresponding to the present Hubble radius is  $k_H = 2\pi S(\eta_0)/\ell_0 = 2\pi\gamma$ . There is another wave number  $k_E = \frac{2\pi S(\eta_E)}{1/H_0} = \frac{k_H}{1+z_E}$ , that its wavelength at the time  $\eta_E$  is the Hubble radius  $1/H_0$ . By matching  $S$  and  $S'/S$  at the joint points, one gets

$$l_0 = \ell_0 b \zeta_E^{-(2+\beta)} \zeta_2^{\frac{\beta-1}{2}} \zeta_s^\beta \zeta_1^{\frac{\beta-\beta_s}{1+\beta_s}}, \quad (14)$$

where  $b \equiv |1 + \beta|^{-(2+\beta)}$ ,  $\zeta_E \equiv \frac{S(\eta_0)}{S(\eta_E)}$ ,  $\zeta_2 \equiv \frac{S(\eta_E)}{S(\eta_2)}$ ,  $\zeta_s \equiv \frac{S(\eta_2)}{S(\eta_s)}$ , and  $\zeta_1 \equiv \frac{S(\eta_s)}{S(\eta_1)}$ . With these specifications, the functions  $S(\eta)$  and  $S'(\eta)/S(\eta)$  are fully determined [1, 4].

The power spectrum of RGWs is defined as

$$\int_0^\infty h^2(k, \eta) \frac{dk}{k} = \langle 0 | h^{ij}(\mathbf{x}, \eta) h_{ij}(\mathbf{x}, \eta) | 0 \rangle. \quad (15)$$

Substituting eq.(3) in eq.(15) with same contribution of each polarization, we get

$$h(k, \eta) = \frac{4l_{pl}}{\sqrt{\pi}} k |h(\eta)|. \quad (16)$$

The spectrum at the present time  $h(k, \eta_0)$  can be obtained, provided the initial spectrum is specified. The initial amplitude of the spectrum is given by

$$h(k, \eta_i) = A \left( \frac{k}{k_H} \right)^{2+\beta}, \quad (17)$$

where the constant  $A$  can be determined by quantum normalization [1], [4], [6]:

$$A = 8\sqrt{\pi} \frac{l_{pl}}{l_0}. \quad (18)$$

It is convenient to consider the amplitude of waves in different range of wave numbers [1, 3]. Thus the amplitude of the spectrum for different ranges are given by [4]

$$h(k, \eta_0) = A \left( \frac{k}{k_H} \right)^{2+\beta}, \quad k \leq k_E, \quad (19)$$

$$h(k, \eta_0) = A \left( \frac{k}{k_H} \right)^{\beta-\gamma} (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_E \leq k \leq k_H, \quad (20)$$

$$h(k, \eta_0) = A \left( \frac{k}{k_H} \right)^{\beta} (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_H \leq k \leq k_2, \quad (21)$$

$$h(k, \eta_0) = A \left( \frac{k}{k_H} \right)^{1+\beta} \left( \frac{k_H}{k_2} \right) (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_2 \leq k \leq k_s, \quad (22)$$

$$h(k, \eta_0) = A \left( \frac{k}{k_H} \right)^{1+\beta-\beta_s} \left( \frac{k_s}{k_H} \right)^{\beta_s} \left( \frac{k_H}{k_2} \right) (1+z_E)^{\frac{-2-\gamma}{\gamma}}, \quad k_s \leq k \leq k_1. \quad (23)$$

The constant  $A$  is determined with the CMB data of WMAP-9 [15]. The observed CMB anisotropies at lower multipoles is  $\Delta T/T \simeq 0.44 \times 10^{-5}$  at  $l \sim 2$  which corresponds to the largest scale anisotropies that have observed so far. Thus taking this to be the perturbations at the Hubble radius gives [16]

$$h(k_H, \eta_0) = A(1+z_E)^{\frac{-2-\gamma}{\gamma}} \simeq 0.44 \times 10^{-5} r. \quad (24)$$

By taking  $k = 2\pi\nu$ , we can obtain  $\nu_E = 1.93 \times 10^{-18}$  Hz,  $\nu_H = 2.28 \times 10^{-18}$  Hz,  $\nu_2 = 9.3 \times 10^{-17}$  Hz and  $\nu_s = 10^8$  Hz [1, 2, 6].

The spectral energy density parameter  $\Omega_g(\nu)$  of gravitational waves is defined through the relation  $\rho_g/\rho_c = \int \Omega_g(\nu) \frac{d\nu}{\nu}$ , where  $\rho_g$  is the energy density of the gravitational waves and  $\rho_c$  is the critical energy density. One reads [1]

$$\Omega_g(\nu) = \frac{\pi^2}{3} h^2(k, \eta_0) \left( \frac{\nu}{\nu_H} \right)^2. \quad (25)$$

Since the space time is assumed to be spatially flat  $K = 0$  with  $\Omega = 1$ , the fraction density of relic gravitational waves must be less than unity,  $\rho_g/\rho_c < 1$ . In order to  $\rho_g/\rho_c$  dose not exceed the level of  $10^{-5}$ , the  $\Omega_g(\nu_1)$  cannot exceed the level of  $10^{-6}$  [1]. Thus based on eq.(25),  $\Omega_g(\nu_1) \simeq 10^{-6}$  for  $\nu_1 \simeq 4 \times 10^{10}$  Hz [6].

The potential  $V(\phi) = \lambda\phi^n$  with scalar field  $\phi$  and constant  $\lambda$  causes inflation. The parameter  $n$  can has the range  $1 < n < 2.1$  [6, 17]. There are three relations that connect the  $r, \beta_s$  and  $\beta$  with  $n$  [6]:

$$r = \frac{8n}{n+2}(1-n_s), \quad (26)$$

$$\beta_s = \frac{4-n}{2(n-1)}, \quad (27)$$

and

$$\beta = -2 - \frac{n}{2(n+2)}(1-n_s), \quad (28)$$

where  $n_s$  is scalar spectral index. And also we can write the  $T_{rh}$  based on the slow-roll inflation as follows [6]

$$T_{rh} = 3.36 \times 10^{-68} \sqrt{\frac{1-n_s}{A_s}} \exp\left[\frac{3(n+2)}{2(1-n_s)}\right], \quad (29)$$

or

$$T_{rh} = 3.36 \times 10^{-68} \sqrt{\frac{1-n_s}{A_s}} \exp\left[\frac{3}{2(1-n_s)} \times \frac{6(1+\beta_s)}{1+2\beta_s}\right], \quad (30)$$

where in the last line we used eq.(27) and  $A_s$  is amplitude of the scalar perturbations.

For taking in account the effect of the  $T_{rh}$  on the  $\zeta_s$  and  $\zeta_1$ , we can consider the following relations [1, 6]:

$$\zeta_s = \left( \frac{\nu_s}{\nu_2} \right) = \frac{S(\eta_2)}{S_{rec}} \frac{S_{rec}}{S(\eta_s)} = \frac{T_{rh}}{T_{CMB}(1+z_{eq})} \left( \frac{g_1}{g_2} \right)^{1/3}, \quad (31)$$

$$\zeta_1 = \left( \frac{\nu_1}{\nu_s} \right)^{(1+\beta_s)} = \frac{S(\eta_s)}{S(\eta_1)} = \frac{m_{pl}}{k_0^p} \left[ \pi A_s (1-n_s) \frac{n}{2(n+2)} \right]^{1/2}$$

$$\times \frac{T_{CMB}}{T_{rh}} \left( \frac{g_2}{g_1} \right)^{1/3} \exp \left[ - \frac{n+2}{2(1-n_s)} \right], \quad (32)$$

where  $S_{rec}$  and  $m_{pl}$  are scale factor at the recombination and Planck mass respectively. The  $g_1 = 200$  and  $g_2 = 3.91$  count the effective number of relativistic species contributing to the entropy during the reheating and recombination respectively [6]. Also we used  $z_{eq} = 3371$ , pivot wave number  $k_0^p = 0.002 \text{ Mpc}^{-1}$  and  $T_{CMB} = 2.718 \text{ K} = 2.348 \times 10^{-13} \text{ GeV}$  from Planck 2015 [8].

When the quantum normalization for the generation of RGWs during inflation is considered, we get

$$\Delta_R(k_0)r^{1/2} = 8\sqrt{\pi}l_{pl}H_0\zeta_1^{\frac{\beta_s-\beta}{1+\beta_s}}\zeta_s^{-\beta}\zeta_2^{\frac{1-\beta}{2}}\zeta_E^{\frac{\beta-1}{\gamma}}\left(\frac{k_0}{k_H}\right)^\beta, \quad (33)$$

where  $k_0 = 2\pi\nu_0 = k_0^p a(\eta_0)$  with  $\nu_0 = 3.09 \times 10^{-18} \text{ Hz}$  [4, 6].

The parameter  $\beta_s$  is as a free parameter if dose not exceed the  $1 + \beta_s > 0$ . The reheating temperature under slow-roll inflation and quantum normalization (eqs.(30, 33)) is very sensitive to the  $\beta_s$ . These conditions are very important for the early universe and we must pay attention to obtain the suitable  $\beta_s$  under these conditions. Otherwise there will be some problems such as discrepancy of  $T_{rh}$  as mentioned in [6] due to some unsuitable  $\beta_s$  like  $\beta_s=1$ . Therefore in case of the slow-roll inflation is correct condition, due to this discrepancy the author in [6] has believed that the quantum normalization in eq.(33) may not be a good initial condition. Note that, if we do not consider quantum normalization, the zero point energy must be removed or the cosmological constant would be 120 orders of magnitude more than observed. So based on this point, we think that the observed difference in  $T_{rh}$  can remove by determining the appropriate parameter  $\beta_s$  in the reheating era. Hence the obtained temperatures based on the calculated  $\beta_s$  are in favour of both slow roll and initial condition of quantum normalization. Thus we will investigate this motivation in the following section.

### III. DETERMINING PARAMETER $\beta_s$

The constant  $A \propto l_0^{-1}$  that is appeared in eq.(18), is determined by quantum normalization. Using eqs.(14, 31, 32) and after straightforward calculation, we obtain  $T_{rh}$  as follows:

$$T_{rh} = \alpha A^{\frac{-(1+\beta_s)}{\beta_s(1+\beta_s)}}, \quad (34)$$



TABLE I: The obtained results for given  $n_s$  and  $A_s$  from WMAP-9 [7]. The  $W, T_s$  and  $T_q$  stand for WMAP, reheating temperature of slow roll and quantum normalization respectively.

<i>Object</i>	$n_s$	$A_s \times 10^9$	$T_s = T_q$ (GeV)	$\beta_s$	$n$	$\beta$	$r$
$W$	$0.972 \pm 0.013$	$2.41 \pm 0.10$	$10^{11}$	5.59	1.24	-2.0054	0.0860
$W + eCMB$	$0.9642 \pm 0.0098$	$2.43 \pm 0.084$	$10^7$	1.2	1.88	-2.0086	0.1389
$W + eCMB + BAO$	$0.9579^{+0.0081}_{-0.0082}$	$2.484^{+0.073}_{-0.072}$	$10^3$	0.63	2.32	-2.0113	0.1811
$W + eCMB + H_0$	$0.9690^{+0.0091}_{-0.0090}$	$2.396^{+0.079}_{-0.078}$	$10^{10}$	2.39	1.51	-2.0067	0.1071
$W + eCMB + BAO + H_0$	$0.9608 \pm 0.008$	$2.464 \pm 0.072$	$10^5$	0.81	2.14	-2.0101	0.1623

where  $\alpha = 3.4 \times 10^6 \times A^{-1.9831}$  is the constant normalization (see Appendix.(A) for more details), therefore

$$T_{rh} = 3.4 \times 10^6 \times A^{-\frac{\beta_s(2.9831+1.9831\beta)+1}{\beta_s(1+\beta)}}. \quad (35)$$

In general it can see that  $A$  is function of  $\beta_s$  based on the eqs.(24, 26, 27) as follows:

$$A = 0.44 \times 10^{-5} \times (1 + z_E)^{\frac{2+\gamma}{\gamma}} \left[ \frac{4(2 + \beta_s)}{3(1 + \beta_s)} (1 - n_s) \right], \quad (36)$$

and also by using eqs.(27, 28), we can get

$$\beta = \frac{\beta_s(n_s - 13) + 2n_s - 14}{6(1 + \beta_s)}. \quad (37)$$

Therefore  $T_{rh}$  in eq.(35) becomes complicated function of  $\beta_s$  due to eqs.(36, 37) for given  $n_s$ .

It seems the observed difference in  $T_{rh}$  that mentioned in the earlier section can remove by determining of the appropriate  $\beta_s$  in eqs.(30, 35) if it dose not exceed the condition  $1 + \beta_s > 0$ , and also the obtained  $\Omega_g$  is not more than level  $\simeq 10^{-6}$  due to  $\beta_s$  [1].

Thus in order to remove the observed difference, we repeat eqs.(30, 35) as follows:

$$T_s = 3.36 \times 10^{-68} \sqrt{\frac{1 - n_s}{A_s}} \exp \left[ \frac{3}{2(1 - n_s)} \times \frac{6(1 + \beta_s)}{1 + 2\beta_s} \right],$$

TABLE II: The obtained results for given  $n_s$  and  $A_s$  from Planck [19]. The  $P, le, W, hl, T_s$  and  $T_q$  stand for Planck, lensing, WMAP, HighL, reheating temperature of slow roll and quantum normalization respectively.

<i>Object</i>	$n_s$	$\ln(10^{10}A_s)$	$T_s = T_q$ (GeV)	$\beta_s$	$n$	$\beta$	$r$
$P$	$0.9616 \pm 0.0094$	$3.103 \pm 0.072$	$10^5$	0.87	2.09	-2.0098	0.1572
$P + le$	$0.9635 \pm 0.0094$	$3.085 \pm 0.057$	$10^6$	1.06	1.96	-2.0090	0.1446
$P + W$	$0.9603 \pm 0.0073$	$3.089^{+0.024}_{-0.027}$	$10^4$	0.77	2.18	-2.0104	0.1657
$P + W + hl$	$0.9585 \pm 0.007$	$3.090 \pm 0.025$	$10^3$	0.67	2.28	-2.0111	0.1769
$P + le + W + hl$	$0.9641 \pm 0.0063$	$3.087 \pm 0.024$	$10^7$	1.13	1.92	-2.0088	0.1407
$P + W + hl + BAO$	$0.9608 \pm 0.0054$	$3.091 \pm 0.025$	$10^5$	0.8	2.15	-2.0102	0.1626

$$T_q = 3.4 \times 10^6 \times A^{-\frac{\beta_s(2.9831+1.9831\beta)+1}{\beta_s(1+\beta)}},$$

where  $T_s$  and  $T_q$  are reheating temperatures based on the slow roll and quantum normalization respectively. In order to find equal temperature  $T_s = T_q$ , we get

$$3.36 \times 10^{-68} \sqrt{\frac{1-n_s}{A_s}} \exp\left[\frac{3}{2(1-n_s)} \times \frac{6(1+\beta_s)}{1+2\beta_s}\right] = 3.4 \times 10^6 \times A^{-\frac{\beta_s(2.9831+1.9831\beta)+1}{\beta_s(1+\beta)}}. \quad (38)$$

Because this relation is complicated function of  $\beta_s$ , we solve it numerically. Then based on the calculated  $\beta_s$ , the corresponding amount of parameters  $r, n$  and  $\beta$  will obtain from eqs.(26, 27, 28) respectively. The obtained results are shown in tables.[I, II] based on WMAP-9 [7] and Planck [19] respectively. Also for more clarity purpose, the diagram of  $T_s$  and  $T_q$  functions are plotted in Fig. [1], panels. (a to e) and Fig.[ 2], panels. (a to f) versus  $\beta_s$  for given parameters based on tables.[I, II] respectively. The red and blue colors

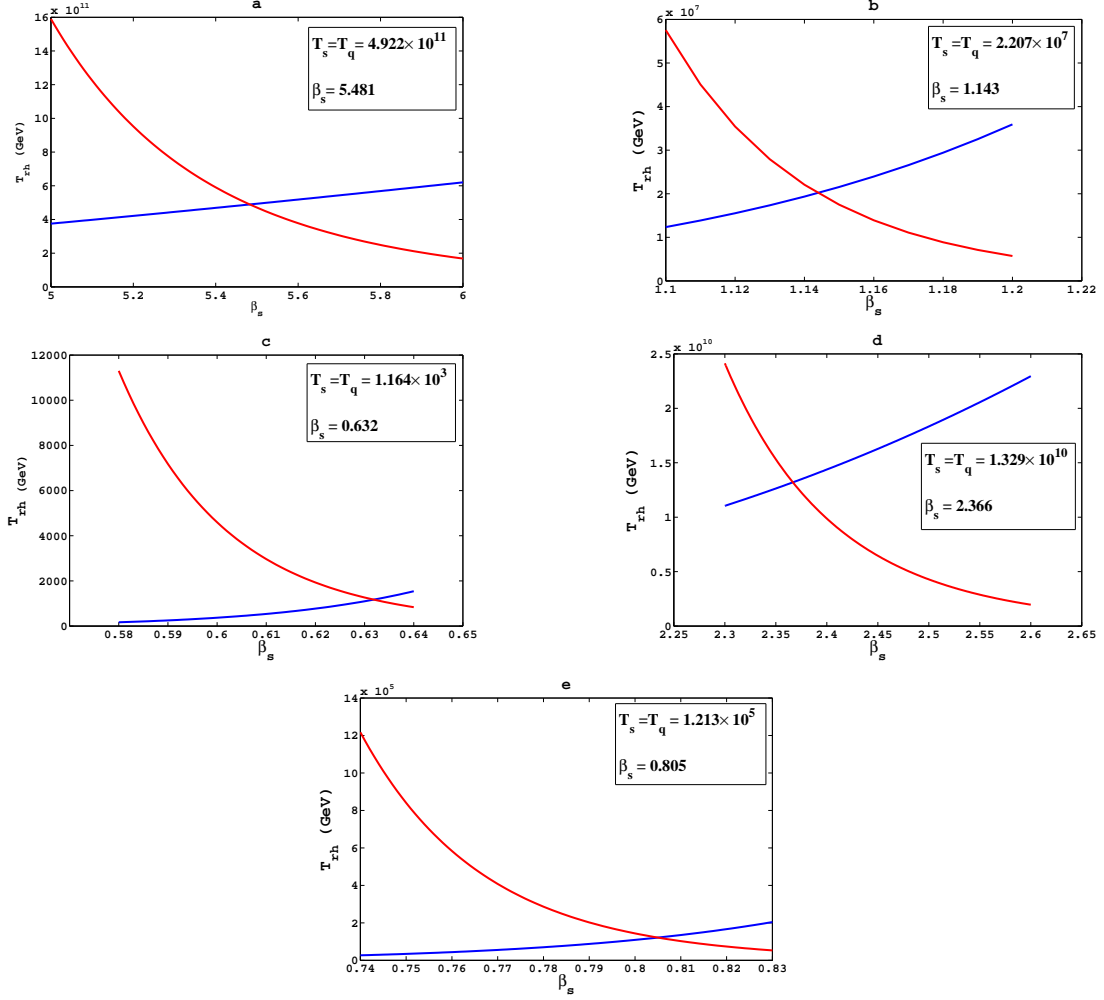


FIG. 1: The obtained results of reheating temperature and  $\beta_s$  for given  $n_s$  and  $A_s$  based on WMAP-9 [7]. The red and blue colors are for  $T_s$  and  $T_q$  respectively.

are for  $T_s$  and  $T_q$  respectively. We have shown for each panel of both figures due to a suitable  $\beta_s$ , there is only one intersection point based on eq.( 38). So by finding the suitable  $\beta_s$ , we can have same reheating temperatures based on the slow roll and quantum normalization and there is no need to bother about the discrepancy of  $T_{rh}$ . Otherwise there will be some problems like discrepancy that mentioned in [6].

Also it is observing that the all amounts of  $T_{rh}$  ( $T_s$  and  $T_q$ ) are in good agreement with the general range  $10 \text{ MeV} \lesssim T_{rh} \lesssim 10^{16} \text{ GeV}$  in both tables for all type of objects. But there are some amounts of  $T_{rh}$  especially in table.[II] that have consistency with the special range  $10^6 \text{ GeV} \lesssim T_{rh} \lesssim 10^9 \text{ GeV}$  based on the DECIGO and BBO detectors [9]. The obtained results of  $\beta_s$  and  $\Omega_g$  do not exceed the their constrains ( $1 + \beta_s > 1, \Omega(\nu_1) \simeq 10^{-6}$ ) in both

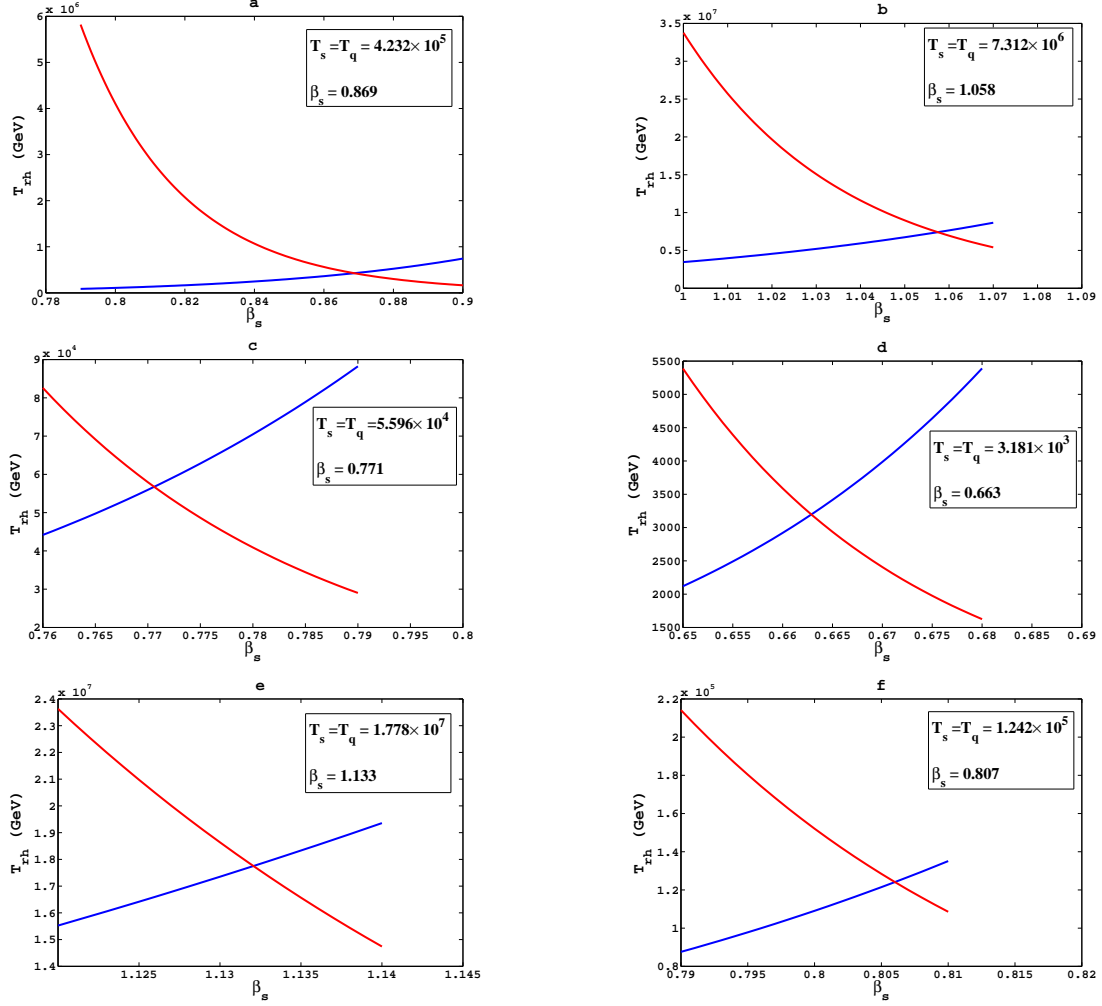


FIG. 2: The obtained results of reheating temperature and  $\beta_s$  for given  $n_s$  and  $A_s$  based on Planck [19]. The red and blue colors are for  $T_s$  and  $T_q$  respectively.

tables for all type of objects. But there are some amounts of  $r$  ( $n$ ) that have consistency with (the range  $1 < n < 2.1$ ) the Planck and WMAP result of  $r \sim 0.1, 0.13$  respectively. The corresponding calculated amounts of  $\beta$  give us better understanding about the uncertainty condition  $1 + \beta < 0$  [4]. The interesting temperature  $\sim 10^7$  GeV found again in the results (see tables). This temperature is best determined for  $r \sim 0.1$  based on the DECIGO and BBO detectors [13].

Therefore this work tells us one can remove the mentioned discrepancy in the  $T_{rh}$  for some specific  $\beta_s$ . As these specific amounts of  $\beta_s$  are in favour of both slow roll and initial condition of quantum normalization. Therefore it may be better to focus on the amounts of  $\beta_s$  that are obtained in this work otherwise there will be some problems like discrepancy.

Thus the obtained results can give us better realization about the history of the universe especially for inflation and reheating stages.

#### IV. CONCLUSION.

The behaviour of the inflation and reheating stages are often known as power law expansion  $S(\eta) \propto \eta^{1+\beta}$ ,  $S(\eta) \propto \eta^{1+\beta_s}$  respectively, with constraints  $1 + \beta < 0, 1 + \beta_s > 0$ . The reheating temperature and  $\beta_s$  are important and give us valuable information about the reheating stage.

It observed that the motivated discrepancy due to considering the slow roll and initial condition of quantum normalization, can remove by determining of the appropriate  $\beta_s$ . Therefore the obtained parameters of  $\beta_s$  were in favour of both conditions. Otherwise there will be some problems like discrepancy of  $T_{rh}$ . The obtained results of  $\beta_s, \beta$  and  $\Omega_g$  did not exceed the their constrains. But there were some amounts of  $r$  ( $n$ ) that have consistency with (the range  $1 < n < 2.1$ ) the Planck and WMAP result of  $r \sim 0.1, 0.13$  respectively. It observed that the all amounts of  $T_{rh}$  ( $T_s$  and  $T_q$ ) were in good agreement with its general range. While some of them were in good agreement with the special range of  $T_{rh}$ . Also it observed the interesting temperature  $\sim 10^7$  GeV that is best determined based on the DECIGO and BBO detectors, found in our results again.

Hence, based on our results we can have better understanding about the history of the universe especially for inflation and reheating stages

#### Appendix A

In order to obtain eq.(34), we can see that constant  $A$  is appeared in eq.(18) like  $A \propto l_0^{-1}$ . By writing eq.(14) in the following form

$$l_0 = m \zeta_s^\beta \zeta_1^{\frac{\beta-\beta_s}{1+\beta_s}}, \quad (\text{A1})$$

where

$$m = \ell_0 b \zeta_E^{-(2+\beta)} \zeta_2^{\frac{\beta-1}{2}}, \quad (\text{A2})$$

we can get for  $A$

$$A \propto \left[ m \zeta_s^\beta \zeta_1^{\frac{\beta-\beta_s}{1+\beta_s}} \right]^{-1}. \quad (\text{A3})$$

On the other hand it is clear that  $\zeta_s \propto T_{rh}$  and  $\zeta_1 \propto T_{rh}^{-1}$  from eqs.(31, 32) respectively. Therefore eq.(A3) will change to the following form

$$A \propto m T_{rh}^{\frac{-\beta_s(1+\beta)}{1+\beta_s}}, \quad (\text{A4})$$

or

$$T_{rh} = \alpha A^{\frac{-(1+\beta_s)}{\beta_s(1+\beta)}}, \quad (\text{A5})$$

where  $\alpha = 3.4 \times 10^6 \times A^{-1.9831}$  is the constant normalization.

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- [1] L.P. Grishchuk, *Lect.Notes Phys.* **562** (2001) 167-194.
  - [2] B. Ghayour and P.K. Suresh, *Class. Quantum Grav.* **29** (2012) 175009.
  - [3] Y. Zhang, *et al.*, *Class. Quantum Grav.* **22** (2005) 1383 -1394.
  - [4] M. Tong, *Class. Quantum Grav.* **29** (2012) 155006.
  - [5] J. Mielczarek, *Phys.Rev.D* **83** (2011) 023502.
  - [6] M. Tong, *Class. Quantum Grav.* **30** (2013) 055013.
  - [7] G. Hinshaw, *et al.*, (2013), [arXiv:1212.5226v3 [astro-ph.CO]].
  - [8] P. A. R. Ade, N. Aghanim Planck Collaboration *et al.*, (2015), [arXiv:1502.01589v2 [astro-ph.CO]].
  - [9] K. Nakayama, *et al.*, *JCAP* **020** (2008) 0806.
  - [10] N. Seto, *et al.*, *Phys. Rev. Lett* **87** (2001) 221103.
  - [11] S. Kawamura, *et al.*, *Class. Quant. Grav.* **23** (2006) S125.
  - [12] C. Cutler, D. E. Holz, *Phys. Rev. D* **80** (2009) 104009.
  - [13] S. Kuroyanagi, *et al.*, *Phys.Rev.D* **84** (2011) 123513.
  - [14] Y. Zhang, *et al.*, *Phys. Rev.D* **81** (2010) 101501.
  - [15] <http://lambda.gsfc.nasa.gov/product/map/current/>
  - [16] Y. Zhang, *et al.*, *Class. Quantum Grav.* **23** (2006) 3783-3800.
  - [17] J. Martin and C. Ringeval, *Phys. Rev.D* **82** (2010) 023511.
  - [18] E. Komatsu, *et al.*, *Astrophys. J. Suppl.* **192** (2011)18.
  - [19] P. A. R. Ade, N. Aghanim Planck Collaboration *et al.*, (2013), [arXiv:1303.5076v2 [astro-ph.CO]].